

## Remote Sensing Image Denoising Using Partial Differential Equations

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**Abstract:** *The presence of noise in images is unavoidable. It may be formed by image formation process, image recording, image transmission etc. These random distortions make it difficult to perform any required image processing task. High quality noise free remote sensing images are necessary for various applications. Therefore, removal of noise is necessary. The purpose of image denoising is to preserve edges as far as possible while removing noise, making the resulting images approximate the ideal image. This paper proposes a method for denoising remote sensing images using a combination of second order and fourth order partial differential equations. The advantages of both second order and fourth order partial differential equations are utilized here. The image is denoised using second order partial differential equations, fourth order partial differential equations and the combination of both. A comparison of the three methods is also presented. The proposed algorithm smooth out more noise and conserve more detail in the denoising process.*

**Keywords:** *Image denoising; Partial differential equation (PDE); Signal to Noise Ratio (SNR); Smoothing.*

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### I. Introduction

Images are a form of data that carries information. As with any other form of data, the information within each picture can be affected by errors or noise. The source of errors varies from image to image, and along with the effects of signal transmission. These errors are responsible for the blurred and deteriorated images. Image denoising techniques improve the quality of an image as perceived by a human. The aim of image denoising is to improve the interpretability of information in images for human viewers, or to provide better input for other automated image processing techniques. These techniques are most useful because many satellite images when examined give inadequate information for image interpretation. The demand for high quality remote sensing images is increasing day by day. There exists a wide variety of techniques for improving image quality. Reduction of noise is essential especially in the field of image processing. Several researchers are continuously working in this direction and provide some good insights, but still there are lot of scope in this field. Noise mixed with image is harmful for image processing. Image denoising play an important role in Image processing task. Remove the noise when the edges are in the preserving state is called image denoising. In the image processing task it is a major and most common problem. If we want a very high quality resolution images as the outcome then we must consider the noise parameters for reducing those parameters to achieve better results. The main purpose or the aim of image denoising is to recover the main image from the noisy image. The proposed thesis introduces a method for image denoising using partial differential equations. Partial differential equations (PDE) denoising method [1] can smooth out the high frequency oscillation while keeping the edges in the high noisy level images.

In recent years many denoising algorithms based on different theories have been proposed. The algorithms include total variation (TV) [2]–[3], wavelets [4]–[5], and nonlocal means [6]. Second order partial differential equations [7]–[8] and fourth order partial differential equations [9]–[10] have been well studied as useful tool for image denoising problem. Fourth order partial differential equations have the disadvantage of speckle effect [10]. In many situations involving multicomponent remotesensing images, a single-component image with a higher SNR or higher spatial resolution is often available. Such an auxiliary image can be used to improve quality of the output image. Although second order partial differential equation based techniques have been demonstrated to be able to achieve a good trade-off between noise removal and edge preservation, they tend to cause blocky effect [10]. The output image looks visually uncomfortable and is likely to cause a computer vision system to falsely recognize as edges the boundaries of different blocks that actually belong to the same smooth area in the original image. It is possible to improve the quality of the output image and preserve more details in the image using an auxiliary image with high SNR [11]. In this case second order partial differential equation is generated from the auxiliary image and this information is used as a reference [12]. Denoising is performed by combining the partial differential equations from the noisy image and the reference image. The similarity of the directions of edges between the noisy image and reference image enables the new algorithm to smooth out more noise and conserve more detail in the denoising process. But such an auxiliary image with high SNR may not be always available. In such cases the algorithm will not be able to remove noise effectively.

This paper proposes use of second order partial differential equations in combination with fourth order partial differential equations to improve the quality of the processed image. Here advantages of both second order partial differential equations and fourth order partial differential equations are utilized. Both the second order partial differential equation and the fourth order partial differential equation models have their strengths and weaknesses depending on the characteristics of the image. The aim is to generate a new solution by taking the best from each of the two methods using a combination. The model can reduce both blocky effect and speckle effect. The model also shows better performance in comparison with second order and fourth order partial differential equation methods. The remainder of this paper is organized as follows. Section II explains about the methodology. Section III discusses the experimental results. Finally, conclusion is given in Section IV.

## II. Methodology

In this section procedure for denoising using second order partial differential equations, fourth order partial differential equations and a combination of both are explained.

### A. Denoising Using Second Order PDE

Second order partial differential equations have been demonstrated to be effective for removing noise and edge preservation. Gradient of the image is utilized here. An image gradient is a directional change in the intensity in an image. Image gradients can be used to extract information from images. Gradient images are created from the original image for this purpose. Each pixel of a gradient image measures the change in intensity of that same point in the original image, in a given direction. To get the full range of direction, gradient images in the x and y directions are computed. One of the most common uses is in edge detection. After gradient images have been computed, pixels with large gradient values become possible edge pixels. The pixels with the largest gradient values in the direction of the gradient become edge pixels, and edges may be traced in the direction perpendicular to the gradient direction.

Let  $u$  denote the image intensity function,  $t$  the time.

$$\frac{\partial u}{\partial t} = \nabla \cdot (g(|\nabla u|)\nabla u)$$

where  $g$  is the diffusion coefficient

The solution of the above equation is equivalent to the minimization of energy functional.

$$E(u) = \int f(|\nabla u|) dx dy$$



Fig.1. Blocky effect

This type PDE can better preserve edges when removing the noise, but its resulting image exhibits serious blocky effect. In this model smooth areas are diffused faster than the other areas. Therefore blocky effects will appear. The effect will increase as time evolves. Therefore the image looks visually uncomfortable and also errors can occur in the computer processing of such an image since false edges may be identified as edges of ideal image. The blocky effect is shown in fig. 1.

### B. Denoising Using Fourth Order PDE

To reduce the blocky effect, fourth order partial differential equations have been introduced. Fourth order partial differential equations can dampen noise much faster than second order partial differential equations. The fourth order partial differential equation model replaces the gradient operator in the second order model with the Laplacian operator. Laplacian is a differential operator given by the divergence of the gradient of a function on Euclidian space. In a Cartesian coordinate system, the Laplacian is given by the sum of second partial derivatives of the function with respect to each independent variable. In image processing and computer vision, the laplacian operator has been used for various tasks.

Let  $u$  denote the image intensity function,  $t$  the time.

$$\frac{\partial u}{\partial t} = -\nabla^2 (g(|\nabla^2 u|)\nabla^2 u)$$

The solution of the above equation is equivalent to the minimization of the energy functional

$$E(u) = \int f(|\nabla^2 u|) dx dy$$

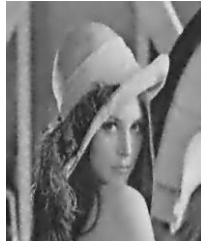


Fig.2. Speckle effect

Fourth order partial differential equations attempt to remove noise and preserve edges by approximating the observed image with a piecewise planar image. In the case of fourth order partial differential equations, the resulting image's smoothness is better than that of second order partial differential equations. But the laplacian operator cannot determine edges. Therefore the fourth-order PDE model blurs edge information. The blocky effect will be reduced and image will look more natural. However, the fourth-order partial differential equation model tends to leave images with speckle artifacts. This is known as speckle effect. The speckle effect is shown in fig.2.

### C. Denoising Using Combination

In the above analysis, we can see that second order PDE model can better preserve image edge, but it easily introduces blocky effect. Fourth order PDE model can better smooth image, but it easily put speckle effect into resulting image and it blurs image edge. Both the second order partial differential equation model and the fourth order partial differential equation model have their strengths and weaknesses. Here advantages of both the second order and fourth order partial differential equations are taken into account. Thus this denoising method can alleviate blocky effect as well as speckle effect effectively and shows better performance.

Let  $u$  denote the image intensity function,  $t$  the time. The proposed model is given by

$$\frac{\partial u}{\partial t} = -wF + (1 - w)H$$

Iteration method to solve the above equation, assuming that image size is  $I \times J$ , where  $F$  and  $H$  are given by,

$$F_{i,j}^n = \frac{\partial^2 g_{1i,j}^n}{\partial x^2} + \frac{\partial^2 g_{2i,j}^n}{\partial y^2}$$

$$H_{i,j}^n = \frac{\partial^2 g_{3i,j}^n}{\partial x} + \frac{\partial^2 g_{4i,j}^n}{\partial y}$$

Where  $w$  is an adjustable parameter and  $i$  and  $j$  are given as  $i=0,1,2,\dots,I$  and  $J=0,1,2,\dots,J$

$$g_1 = C(|u_{xx}|)u_{xx}$$

$$g_2 = C(|u_{yy}|)u_{yy}$$

$$g_3 = \frac{u_x}{\sqrt{u_x^2 + u_y^2 + 1}}$$

$$g_4 = \frac{u_y}{\sqrt{u_x^2 + u_y^2 + 1}}$$

The proposed model is compared with the second order and fourth order partial differential equation methods Simulation results shows that the proposed model gives better performance effectively removing noise. The model can alleviate blocky effect and speckle effect simultaneously and results in a more natural image.

#### D. Performance Measures

The performance of the three denoising methods are measured on the basis of performance parameters Peak Signal to Noise Ratio and Signal to Noise Ratio. SNR compares the level of a desired signal to the level of background noise. It is defined as the ratio of signal power to the noise power, often expressed in decibels. PSNR represents a measure of the peak error. It is the ratio between the maximum possible power of a signal and the power of corrupting noise.

Signal to Noise Ratio (SNR) is given by

$$SNR = \frac{\text{Variance of image}}{\text{Variance of noise}}$$

Peak Signal to Noise Ratio (PSNR) is given by

$$PSNR = 10\log_{10}\left(\frac{255 \times 255}{\sum_{ij}(u_{ij} - v_{ij})^2}\right)$$

where u is the original image and v is the compared image.

### III. Result And Discussion

Ideal input image is shown in the figure. Gaussian noise with zero mean and standard deviation 20 is added. The image is then denoised using second order partial differential equation, fourth order partial differential equation and the proposed combination of partial differential equations.



Fig.3. Ideal Image



Fig.4. Noisy Image



Fig.5. Denoised Image using 2<sup>nd</sup> order PDE



Fig.6. Denoised Image Using 4<sup>th</sup> order PDE



Fig.7. Denoised Image Using Proposed Method

Table I gives a comparison of SNR and PSNR of the three methods for different noise values. Noise with standard deviation 15, 20 and 25 are added to the input ideal image to degrade the image. The results shows that the proposed method has better SNR and PSNR than the other methods and exhibits better results.

**Table I. Comparison Of Denoising Methods**

Method		2 <sup>nd</sup> Order Model	4 <sup>th</sup> Order Model	Proposed Model
SD				
15	SNR	6.2892	6.1314	7.3994
	PSNR	23.3348	22.9880	24.3573
20	SNR	6.4475	6.2139	7.5020
	PSNR	23.1706	22.7641	24.3149
25	SNR	6.1871	5.9751	7.3581
	PSNR	22.4751	22.0904	23.9976

Noise with different standard deviation is added to the image for simulation. When noise with standard deviation 20 is added, the second order partial differential equation denoising model gives SNR value of 6.4475 and PSNR value of 23.1706. The fourth order partial differential equation denoising model gives SNR value of 6.2139 and PSNR value of 22.7641. The proposed partial differential equation denoising model gives SNR value of 7.5020 and PSNR value of 24.3149. The same experiment is done using noise with standard deviation 15 and 10. It is clear that in all cases the proposed model gives better results.

#### IV. Conclusion

In this thesis, I have presented a denoising method for remote sensing images using a combination of second order and fourth order partial differential equations. The proposed algorithm utilizes the advantages of both second order and fourth order partial differential equations. The algorithm smooths out noise by conserving more detail of the image in the denoising process. The existing method using second order partial differential equations has the disadvantage of blocky effect and that using fourth order partial differential equations has the disadvantage of speckle effect. The proposed algorithm gives better results eliminating the blocky effect and speckle effect. We have denoised the image using second order, fourth order partial differential equations and using the proposed method. A comparison of the three methods is also done using SNR and PSNR parameters for different noise values. The proposed method gives better values.

#### References

- [1]. Guy Gilboa, Nir Sochen, and Yehoshua Y. Zeevi, "Estimation of Optimal PDE - Based Denoising In the SNR Sense" IEEE Transactions on image processing, vol.15, no. 8, August 2006.
- [2]. L. Rudin, S. Osher, and E. Fatemi, "Nonlinear total variation based noise removal algorithms," Phys. D, vol. 60, no. 1-4, pp. 259-268, Nov. 1992.
- [3]. A. Chambolle, "An algorithm for total variation minimization and applications," J. Math. Imag. Vis., vol. 20, no. 1/2, pp. 89-97, Jan. 2004.
- [4]. F. Abramovitch, T. Sapatinas, and B.W. Silverman, "Wavelet Thresholding via a Bayesian approach," J. R. Statist. Soc. Ser. B, vol. 60, no. 4, pp. 725-749, 1998.
- [5]. J. Portilla, V. Strela, M. J. Wainwright, and E. P. Simoncelli, "Image denoising using scale mixtures of gaussians in the wavelet domain," IEEE Trans. Image Process., vol. 12, no. 11, pp. 1338-1351, Nov. 2003.
- [6]. A. Buades, B. Coll, and J.-M. Morel, "A non-local algorithm for image denoising" in Proc. IEEE Conf. Comput. Vis. Pattern Recognit., 2005, pp. 60-65.
- [7]. P. Perona and J. Malik, "Scale-space and edge detection using anisotropic diffusion" IEEE Transactions on Pattern Analysis and Machine Intelligence vol. 12, no. 7, pp. 629-639, 1990.
- [8]. L. You, W. Xu, A. Tannenbaum, and M. Kaveh, "Behavioral analysis of anisotropic diffusion in image processing" IEEE Trans. Image Processing vol. 5, pp. 1539-1553, Nov. 1996.
- [9]. J. B. Greer and A. L. Bertozzi, "Traveling wave solutions of fourth order PDEs for image processing," SIAM Journal on Mathematical Analysis, vol. 36, no. 1, pp. 38-68, 2005.
- [10]. Yu-Li You and M. Kaveh, "Fourth-Order Partial Differential Equations for Noise Removal," IEEE Trans. Image Process., vol. 9, no. 10, October 2000.
- [11]. P. Scheunders and S. De Backer, "Wavelet denoising of multicomponent images using gaussian scale mixture models and a noise-free images as image processing," IEEE Trans. Image Process., vol. 16, no. 7, pp. 1865-1872, Jul. 2007.
- [12]. Peng Liu, Fang Huang, Guoqing Li, and Zhiwen Liu, "Remote sensing image denoising using partial differential equations and auxiliary images as priors," IEEE geoscience and remote sensing letters, vol. 9, no. 3, May 2012, pp. 358-362.